REFLECTOR ANTENNAS
If parallel rays are incident upon a parabolic reflecting surface, it converges at its focus.

There are different constructions of parabolic reflectors:
- Front fed (single reflector) reflector antenna system
- Cassegrain and Gregorian antenna system
- Offset reflector antenna system

The feed can be WG, horn, corrugated horn, and array of feeds.

The corrugated one is the best for low X-polarizations and to match the field produced by the reflector at the focus.

1. Parabolic reflector

Front fed reflector ant. system

Cassegrain

Parabolic subreflector

Hyperbolic subreflector

Feed

Offset reflector ant. system
Fan beam antenna
Antenna command for

Command Guided Missile
At conical scan the antenna traces a cone pattern around its central axis. Used in tracking radars with target azimuth and elevation being taken from the mechanical position of the antenna.

Conical scan antenna
2-Single parabolic reflector

Geometry and surface equation

The design is based on optical geometry techniques (ignoring diffractions from the rim of the reflector), by choosing a plane perpendicular to the axis of the reflector through the focal point

\[
OP + PQ = 2f = \text{cons.}
\]

\[
OP = r', \quad PQ = r' \cos \theta
\]

\[
OP + PQ = r' + r' \cos \theta = r'(1 + \cos \theta')
\]

\[
\therefore r' = \frac{2f}{(1 + \cos \theta')} = f \cdot \sec^2 \left( \frac{\theta'}{2} \right)
\]

or

\[
f = r' \cos^2 \left( \frac{\theta'}{2} \right)
\]
Note that the symmetry of the parabolic imposes no variation to the angle \((\phi)\) and so we replace the spherical coordinates to the rectangular coordinates by:

\[
r' + r' \cos \theta = \sqrt{x'^2 + y'^2 + z'^2} + z' = 2f
\]

\[
\therefore \sqrt{x'^2 + y'^2 + z'^2} = 2f - z'
\]

\[
x'^2 + y'^2 + z'^2 = 4f^2 + z'^2 - 4fz'
\]

\[
\therefore x'^2 + y'^2 = 4f(f - z')
\]

Another approach relates \((\theta_o)\) is:

\[
\theta_o = \tan^{-1}\left(\frac{d / 2}{z_o}\right)
\]

\[
z_o = f - \frac{x_o^2 + y_o^2}{4f} = f - \frac{(d / 2)^2}{4f} = f - \frac{d^2}{16f} \quad \text{[for \quad x_o^2 + y_o^2 \leq (d / 2)^2]}
\]

\[
\therefore \theta_o = \tan^{-1}\left(\frac{d / 2}{f - \frac{d^2}{16f}}\right) = \tan^{-1}\left(\frac{f}{2d}\right) \quad \text{or} \quad f = \left(\frac{d}{4}\right) \cot\left(\frac{\theta_o}{2}\right)
\]
Method of analysis

1-Induced current density:

Current density $J_s$ on the parabolic reflector surface:

$$\bar{J}_s = \hat{n}x\bar{H} = \hat{n}x(\bar{H}_i + \bar{H}_r)$$

Using the approximation that the surface is an infinite plane surface (image theory), we get:

$$\hat{n}x\bar{H}_i = \hat{n}x\bar{H}_r$$

where $H_i$ and $H_r$ represent, respectively, the incident and reflected magnetic field components evaluated at the surface of the conductor, and $n$ is a unit vector normal to the surface.

Use the physical optics approximation (curvature of the rays and the dish $\gg \lambda$), the induced current density $J_s$ is formulated over the illuminated side of the reflector ($S_i$)

$$\therefore \bar{J}_s = \hat{n}x\bar{H} = \hat{n}x(\bar{H}_i + \bar{H}_r) = 2\hat{n}x\bar{H}_i = 2\hat{n}x\bar{H}_r$$

This current density is then integrated over the surface of the reflector to yield the far zone radiation fields.

More accurate, valid for wide angle but it takes more time for computation.
2-Aperture distribution method:
The field reflected by the surface of the parabolic is found over a plane which is normal to the axis of the reflector. Geometrical optics techniques are usually employed to accomplish this. The plane taken is through the focal point and is designated as the **aperture plane**
Equivalent current sources are formed over that plane and zero outside the projected area
This current density is then integrated over the surface of the aperture plane to yield the far zone radiation fields

Simple,
Less computational time
But valid only for narrow angle around the axis
3-Cross polarization

The field reflected by the parabolic have both X, Y components, while the incident had only Y component. Thus the Y component is called the co-polarized (principal polarization) while X component is called the cross polarization.

**Polarization purity**

For very narrow beam reflectors or for angles near the boresight axis ($\theta \approx 0^\circ$), the cross-polarized $x$-component diminishes and it vanishes on axis ($\theta = 0^\circ$).

The smaller $f/d$ ratio, the larger the subtended angle $2\theta_o$, the smaller the cross polarization i.e., better polarization purity.
4-Directivity and effective aperture

\[ A_{em} = \frac{\lambda^2}{4\pi} D_0 \Rightarrow D_0 = \frac{4\pi}{\lambda^2} A_{em} \]

\[ \varepsilon_{ap} = \frac{A_{em}}{A_p} \Rightarrow A_{em} = \varepsilon_{ap} A_p \]

\[ D_0 = \frac{4\pi}{\lambda^2} \varepsilon_{ap} A_p = \varepsilon_{ap} \left( \frac{4\pi}{\lambda^2} A_p \right) (D_0)_{\text{max}} \]

\[ = \varepsilon_{ap} \left( \frac{\pi d/\lambda}{2} \right)^2 \]

\( \varepsilon_{ap} \) is the aperture efficiency, depends on the feed pattern and \((f/d)\) or the subtended angle \(2\theta_0\)

Where:

\( \varepsilon_s \) = spillover efficiency

\( \varepsilon_t \) = taper efficiency

\( \varepsilon_p \) = phase efficiency

\( \varepsilon_x \) = polarization efficiency

\( \varepsilon_b \) = blockage efficiency

\( \varepsilon_r \) = surface random error efficiency

Optimum value is 0.81
The **aperture efficiency** is generally the product of the

1. fraction of the total power that is radiated by the feed, intercepted, and collimated by the reflecting surface (generally known as spillover efficiency)

2. uniformity of the amplitude distribution of the feed pattern over the surface of the reflector (generally known as taper efficiency)

3. phase uniformity of the field over the aperture plane (generally known as phase efficiency)

4. polarization uniformity of the field over the aperture plane (generally known as polarization efficiency)

5. blockage efficiency

6. random error efficiency over the reflector surface
**Phase error**

Any departure of the phase, over the aperture of the antenna, from uniform can lead to a significant diminution of its directivity.

For a paraboloidal reflector system the **phase errors result from**

1. displacement (defocusing) of the feed phase center from the focal point

2. Deviation of the reflector surface from a parabolic shape or random errors at the surface of the reflector

3. Departure of the feed wave fronts from spherical shape
4. Cassegrain reflector

Geometry

1. Larger (main) reflector is parabola or parabola of revolution (paraboloid)
2. Smaller (secondary) reflector is hyperbola or hyperbola of revolution (hyperboloid)

For a 2-reflector system with high magnification:
(large ratio of main reflector $D$ to subreflector $d$)

1. Amplitude distribution is primarily controlled by subreflector curvature
2. Phase distribution is primarily controlled by main reflector curvature
Method of analysis

1-Virtual feed

The real feed and subreflector are replaced by a virtual feed located at the focus of the main reflector.

The virtual feed and main reflector is analyzed by either aperture or current distribution method.
2-Equivalent parabola

- \( f_m \): the focal length of the main reflector
- \( A \): the focus of the main reflector
- \( B \): the focus of the Cassegrain and equivalent parabola
- \( f_e \): the focal length of the equivalent parabola

The main dish and the subreflector are replaced by an equivalent parabola located at a certain distance from the real feed.

The equivalent is obtained from the intersection points of the extension of the incident ray on the subreflector and the reflected ray from the main reflector.
Advantages and disadvantages

1. Ability to place the feed in a convenient location
2. Reduction of spillover and minor lobe radiation
3. Ability to obtain an equivalent focal length much greater than physical length
4. Capability for scanning and/or broadening of the beam by moving one of the reflecting surfaces
5. lower X-polar level

1- Blockage by subreflector
2- Nonuniform excitation current
3- Diffraction from subreflector edges