Lecture # 9

- Filter Design By The Insertion Loss Method
  - Characterization by Power Loss Ratio
    - Maximally Flat
    - Equal Ripple
  - Design Steps
    - Low-pass prototype design.
    - Scaling and conversion.
    - Implementation.
      - Using Stubs.
      - Using High-Low Impedance Sections.

Characterization by Power Loss Ratio

Maximally Flat

\[ P_{LR} = 1 + k^2 \left( \frac{\omega}{\omega_c} \right)^{2N} \]

Equal Ripple

\[ P_{LR} = 1 + k^2 T_N^2 \left( \frac{\omega}{\omega_c} \right) \]

Linear Phase

\[ \phi(\omega) = A \omega \left[ 1 + p \left( \frac{\omega}{\omega_c} \right)^{2N} \right] \]

Elliptic Function

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The process of filter design using IL method

Maximally Flat Low-Pass Filter Prototype

For a 2 elements prototype. (N=2) Assume a source impedance of 1 Ω. Cutoff frequency $\omega_c=1$.

$$P_{LR} = 1 + \omega^4$$

$$Z_{in} = j\omega L + \frac{R(1 - j\omega RC)}{1 + \omega^2 R^2 C^2}$$

$$P_{LR} = \frac{1}{1 - \left|1\right|^2} = \frac{1}{1 - (Z_{in} - 1)(Z_{in} + 1)[(Z_{in}^* - 1)(Z_{in}^* + 1)]} = \frac{\left|Z_{in} + 1\right|^2}{2(Z_{in} + Z_{in}^*)}$$

$$Z_{in} + Z_{in}^* = \frac{2R}{1 + \omega^2 R^2 C^2}$$

$$\left|Z_{in} + 1\right|^2 = \left(\frac{R}{1 + \omega^2 R^2 C^2} + 1\right)^2 + \left(\omega L - \frac{\omega CR^2}{1 + \omega^2 R^2 C^2}\right)^2$$

$$P_{LR} = \frac{1 + \omega^2 R^2 C^2}{4R} \left[\left(\frac{R}{1 + \omega^2 R^2 C^2} + 1\right)^2 + \left(\omega L - \frac{\omega CR^2}{1 + \omega^2 R^2 C^2}\right)^2\right]$$
Maximally Flat Low-Pass Filter Prototype

\[ P_{LR} = \frac{1}{4R} \left( R^2 + 2R + 1 + R^2 \omega^2 C^2 + \omega^2 L^2 + \omega^4 L^2 C^2 R^2 - 2\omega^2 LCR^2 \right) \]

\[ P_{LR} = 1 + \frac{1}{4R} \left( (1-R)^2 + \left( R^2 C^2 + L^2 - 2LCR^2 \right) \omega^2 + \omega^4 L^2 C^2 R^2 \right) \]

\[ \omega = 0 \quad \Rightarrow \quad P_{LR} = 1 + (0)^4 = 1 \quad \Rightarrow \quad P_{LR} = 1 + \frac{1}{4R} \left( (1-R)^2 \right) \quad \text{R} = 1 \]

\[ C^2 + L^2 + 2LC = 0 \]

\[ L = C \]

Note: The coefficient of the \( \omega^2 \) should be zero

\[ P_{LR} = 1 + \omega^4 = 1 + (0)\omega^2 + \omega^4 \]

Note: The coefficient of the \( \omega^4 \) should be 1

\[ \frac{1}{4} L^2 C^2 = \frac{1}{4} L^4 = 1 \]

\[ L = C = \sqrt{2} \]

Maximally Flat Low-Pass Filter Prototype

For an arbitrary number of elements \( N \).

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Maximally Flat Low-Pass Filter Prototype

\[ g_o = 1.0 = \begin{cases} \text{generator resistance} \\ \text{generator conductance} \end{cases} \]

\[ g_k = 2 \sin \left( \frac{(2k-1)\pi}{2N} \right) = \begin{cases} \text{inductance for series inductance} \\ \text{capacitance for shunt capacitors} \end{cases} \]

\[ g_{N+1} = 1.0 = \begin{cases} \text{load resistance if } g_N \text{ is a shunt capacitor} \\ \text{load conductance if } g_N \text{ is a series inductor} \end{cases} \]

Element Values of Maximally Flat Low-Pass Filter Prototype

\begin{array}{ccccccccccc}
N & g_1 & g_2 & g_3 & g_4 & g_5 & g_6 & g_7 & g_8 & g_9 & g_{10} \\
1 & 2.0000 & 1.0 & & & & & & & & \\
2 & 1.4142 & 1.4142 & 1.0 & & & & & & & \\
3 & 1.0000 & 2.0000 & 1.0000 & 1.0 & & & & & & \\
4 & 0.7654 & 1.8478 & 1.8478 & 0.7654 & 1.0 & & & & & \\
5 & 0.6180 & 1.6180 & 2.0000 & 1.6180 & 0.6180 & 1.0 & & & & \\
6 & 0.5176 & 1.4142 & 1.9318 & 1.9318 & 1.4142 & 0.5176 & 1.0 & & & \\
7 & 0.4450 & 1.2470 & 1.8019 & 2.0000 & 1.8019 & 1.2470 & 0.4450 & 1.0 & & \\
8 & 0.3902 & 1.1111 & 1.6629 & 1.9616 & 1.9616 & 1.6629 & 1.1111 & 0.3902 & 1.0 & \\
9 & 0.3473 & 1.0000 & 1.5321 & 1.8794 & 2.0000 & 1.8794 & 1.5321 & 1.0000 & 0.3473 & 1.0 \\
\end{array}

Element Values of Maximally Flat Low-Pass Filter Prototype \( (g_o=1, \omega_c=1, N=1 \text{ to } 9) \)
Maximally Flat Low-Pass Filter Prototype

- To determine the order (size) of the filter. Is usually determined by the specification of the insertion loss at some frequency in the stopband of the filter.

\[
\text{Cutoff frequency } \omega_c = 1.0.
\]

\[
P_{LR} = 1 + k^2 T_N^2 \left( \frac{\omega_c}{\omega_c^2 - 1} \right)
\]

For the Chebyshev polynomials

\[
T_N(0) = \begin{cases} 
0 & \text{for } N \text{ odd} \\
1 & \text{for } N \text{ even}
\end{cases}
\]

Since

\[
T_2(x) = 2x^2 - 1
\]

\[
1 + k^2 (2\omega_c^2 - 1)^2 = 1 + k^2 (4\omega_c^4 - 4\omega_c^2 + 1)
\]

\[
1 + k^2 (4\omega_c^4 - 4\omega_c^2 + 1) = 1 + \frac{1}{4R} \left( (1 - R)^2 + (R^2C^2 + L^2 - 2LCR^2)\omega_c^2 + L^2C^2R^4\omega_c^4 \right)
\]
**Equal-Ripple Low-Pass Filter Prototype**

Note:
\[
\omega = 0 \quad \rightarrow \quad 1 + k^2 = 1 + \frac{(1-R)^2}{4R} \quad \Rightarrow \quad R = 1 + 2k^2 \pm 2k\sqrt{1+k^2} \quad \text{N is even}
\]

Note: The coefficient of the \(\omega^2\) and \(\omega^4\) should same for both sides
\[
4k^2 = \frac{1}{4R} L^2 C^2 R^2 \\
-4k^2 = \left( R^2 C^2 + L^2 - 2LCR^2 \right) \quad \text{Solved together to find a value for both L & C}
\]

Since \(R\) is not equal to one, a quarter wavelength matching section can be used.

Note: for odd \(N\), \(R\) is equal to 1.

---

**Equal-Ripple Low-Pass Filter Prototype**

\(g_o = 1.0\)

\[
g_i = \frac{2}{\gamma} \sin \left( \frac{\pi}{2N} \right) \quad \text{for} \quad k = 1, 2, 3, \ldots N
\]

\[
g_i = \frac{1}{g_{i-1}} \frac{4\sin \left( \frac{(2k-1)\pi}{2N} \right) \cdot \sin \left( \frac{(2k-3)\pi}{2N} \right)}{\gamma^2 + \sin^2 \left( \frac{(k-1)\pi}{N} \right)} \quad \text{for} \quad k = 1, 2, 3, \ldots N
\]

\[
g_{N+1} = \begin{cases} 
  1.0 & \text{for N odd} \\
  \coth^2 \left( \frac{\beta}{4} \right) & \text{for N even}
\end{cases}
\]

\[
\beta = \ln \left( \coth \left( \frac{G_r}{17.37} \right) \right)
\]

\[
\gamma = \sinh \left( \frac{\beta}{2N} \right)
\]

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Equal-Ripple Low-Pass Filter Prototype

\[ N \quad g_1 \quad g_2 \quad g_3 \quad g_4 \quad g_5 \quad g_6 \quad g_7 \quad g_8 \quad g_9 \quad g_{10} \quad g_{11} \]

\[
\begin{array}{cccccccccc}
1 & 0.6986 & 1.0000 & & & & & & & \\
2 & 1.4029 & 0.7071 & 1.9841 & & & & & & \\
3 & 1.5903 & 1.0000 & 2.2145 & 1.0000 & & & & & \\
4 & 6.6703 & 1.1926 & 2.3961 & 0.8449 & 1.9841 & & & & \\
5 & 1.7058 & 1.2296 & 2.5438 & 1.2296 & 1.7058 & 1.0000 & & & \\
6 & 1.7254 & 1.2479 & 2.6064 & 1.3137 & 2.4758 & 0.8996 & 1.9841 & & \\
7 & 1.7372 & 1.2583 & 2.6381 & 1.3444 & 2.6381 & 1.2463 & 1.7472 & 1.0000 & \\
8 & 1.7451 & 1.2647 & 2.6664 & 1.3590 & 2.6984 & 1.3389 & 2.5093 & 0.8796 & 1.9841 \\
9 & 1.7504 & 1.2690 & 2.6878 & 1.3653 & 2.7239 & 1.3673 & 2.6678 & 1.2690 & 1.7504 & 1.0000 \\
10 & 1.7543 & 1.2721 & 2.7094 & 1.3715 & 2.7392 & 1.3806 & 2.7253 & 1.3485 & 2.5239 & 0.8842 & 1.8841 \\
\end{array}
\]

Element Values for Equal-Ripple Low-pass filter prototypes
\((g_o=1, \omega_c=1, N=1\text{ to }10, 0.5\text{ dB ripple})\)

\[
\text{Equal-Ripple Low-Pass Filter Prototype}
\]

\[
\begin{array}{cccccccccccc}
N & g_1 & g_2 & g_3 & g_4 & g_5 & g_6 & g_7 & g_8 & g_9 & g_{10} & g_{11} \\
\hline
1 & 2.9853 & 1.0000 & & & & & & & & & \\
2 & 3.1013 & 0.5339 & 5.8095 & & & & & & & & \\
3 & 3.3487 & 0.7117 & 3.3487 & 1.0000 & & & & & & & \\
4 & 3.4589 & 0.7483 & 4.3471 & 0.5920 & 5.8095 & & & & & & \\
5 & 3.4817 & 0.7018 & 4.5381 & 0.7678 & 3.4817 & 1.0000 & & & & & \\
6 & 3.5045 & 0.7685 & 4.6061 & 0.9729 & 4.6431 & 0.6033 & 5.8095 & & & & \\
7 & 3.3182 & 0.7723 & 4.6386 & 0.8029 & 4.6386 & 0.7723 & 3.5182 & 1.0000 & & & \\
8 & 3.5277 & 0.7045 & 4.6575 & 0.8089 & 4.6990 & 0.8018 & 4.4990 & 0.6072 & 5.8095 & & \\
9 & 3.5340 & 0.7760 & 4.6692 & 0.8118 & 4.7272 & 0.8118 & 4.6692 & 0.7760 & 3.5340 & 1.0000 & \\
10 & 5.5384 & 0.7771 & 4.6768 & 0.8136 & 4.7425 & 0.8164 & 4.7260 & 0.8051 & 4.5142 & 0.6991 & 5.8095 & \\
\end{array}
\]

Element Values for Equal-Ripple Low-pass filter prototypes
\((g_o=1, \omega_c=1, N=1\text{ to }10, 3\text{ dB ripple})\)

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Equal-Ripple Low-Pass Filter Prototype

Attenuation versus normalized frequency for equal-ripple filter prototypes. 0.5 dB ripple level.

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Equal-Ripple Low-Pass Filter Prototype

Attenuation versus normalized frequency for equal-ripple filter prototypes. 3 dB ripple level.

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Filter Transformation

- The low-pass prototypes of the previous sections were normalized designs having a source impedance of \( R_s = 1 \, \Omega \) and a cutoff frequency of \( \omega_c = 1 \). We can do:

1. Impedance Scaling.
2. Frequency Scaling.
3. Converting to give:
   - High-pass.
   - Band-pass.
   - Band-stop.

Impedance Scaling

If \( R_o \) is the source impedance. (=1 in the prototype)

\[ L, C \text{ & } R_o \] are the component values for the Original prototype

\[ L', C' \text{ & } R'_o \] are the impedance scaled values.

\[ Z = j\omega L' = zR' = j\omega LR_o \]

\[ Z = \frac{1}{j\omega C'} = zR' = \frac{R_o}{j\omega C'} \]

\[ R'_o = R_o \]
\[ L' = R_o L \]
\[ C' = \frac{C}{R_o} \]
\[ R'_L = R_o R_L \]
**Frequency Scaling (Low Pass Filters)**

Cutoff frequency changes from unity to $\omega_c$. 

Then the new power loss ratio will be 

$$ P'_{LR}(\omega) = P_{LR}\left(\frac{\omega}{\omega_c}\right) $$

Cutoff will occur at $\omega = \omega_c$ instead when $\omega = 1$

The prototype elements values are $\frac{j\omega L_k}{\omega_c}$ and $\frac{1}{j\omega C_k}$

The new elements values are determined by:

$$ jX_k = j\frac{\omega}{\omega_c}L_k = j\omega L'_k $$

$$ L'_k = \frac{L_k}{\omega_c} $$

$$ jB_k = j\frac{\omega}{\omega_c}C_k = j\omega C'_k $$

$$ C'_k = \frac{C_k}{\omega_c} $$

Both Frequency and Impedance Scaling

$$ L'_k = \frac{R_k L_k}{\omega_c} \quad C'_k = \frac{C_k}{R_k \omega_c} $$

**Impedance and Frequency Scaling (high Pass Filters)**

(a) Low-pass filter prototype response for $\omega_c = 1$.

(b) Frequency scaling for low-pass response.

(c) Transformation to high-pass response.

Because we need high pass we multiply by (-)

$$ P_{LR} = 1 + k^2\left(\frac{\omega}{\omega_c}\right)^{2N} $$

$\omega \longleftrightarrow \frac{\omega}{\omega_c}$

$\omega = 0 \longleftrightarrow \omega = \pm \infty$
Impedance and Frequency Scaling (high Pass Filters)

\[ jX_k = -j \frac{\omega}{\omega} L_k = \frac{1}{j \omega C'_k} \]  
\[ jB_k = -j \frac{\omega}{\omega} C_k = \frac{1}{j \omega L'_k} \]

(series inductors will be replaced with capacitors)

(shunt capacitors will be replaced with inductors)

\[ C'_k = \frac{1}{\omega L_k} \quad L'_k = \frac{1}{\omega C_k} \]

Both Frequency and Impedance Scaling

\[ C'_k = \frac{1}{R \omega L_k} \quad L'_k = \frac{R}{\omega C_k} \]

Example 8.3

Design a maximally flat low-pass filter with a cutoff frequency of 2 GHz, impedance of 50 \( \Omega \), and at least 15 dB insertion loss at 3 GHz. Compute and plot the amplitude response and group delay for \( f = 0 \) to 4 GHz, and compare with an equal-ripple (3.0 dB ripple) and linear phase filter having the same order.

Answer:

\[ \omega_c = 2 \text{ GHz} \]

At 3 GHz need 15 dB IL

\[ \left| \frac{\omega}{\omega_c} \right| - 1 = \left| \frac{3}{2} \right| - 1 = 0.5 \]
Example 8.3

\[ N = 5 \]

\[ \omega = 2 \text{ GHz} \]

\[ R_v = 50 \Omega \]

\[ g_1 = 0.618 \]

\[ g_2 = 1.618 \]

\[ g_3 = 2.000 \]

\[ g_4 = 1.618 \]

\[ g_5 = 0.618 \]

\[ C_1' = \frac{C_1}{R_v \omega_c} = \frac{g_1}{R_v \omega_c} = \frac{0.618}{50 \times \pi \times 2 \times 10^9} = 9836 \times 10^{-13} \text{ pF} \]

\[ L_2' = 6.438 \text{ nH} \]

\[ C_3' = 3.183 \text{ pF} \]

\[ L_4' = 6.438 \text{ nH} \]

\[ C_5' = 0.984 \text{ pF} \]
Example 8.3

Amplitude response.

Example 8.3

(b) Group delay response.
**Bandpass and Bandstop Transformations**

### Bandpass

\[
\frac{\omega}{\omega_o} = \frac{\omega_o - \frac{\omega_o}{\omega_o}}{\omega_o - \omega_o} = \frac{1}{\frac{\omega_o - \omega_o}{\omega_o - \omega_o}} = \frac{\omega_o - \omega_o}{\omega_o - \omega_o} = 1
\]

\[
\Delta = \frac{\omega_o - \omega_o}{\omega_o - \omega_o} = \frac{\omega_o - \omega_o}{\omega_o - \omega_o}
\]

\[
\omega_o = \sqrt{\omega_o \omega_o}
\]

### Transformation to Band-pass Response

- **Low-pass filter prototype response for \( \omega_c \)**
- **Transformation to band-pass response**
  - \( \omega = \omega_o \) → \( P_{LR} = 1 \)
  - \( \omega = 1 \) → \( P_{LR} = 2 \)

### Transformation to Band-stop Response

- **Transformation to band-stop response**
  - \( \omega = \omega_o \) → \( P_{LR} = 1 \)
  - \( \omega = \omega_c, \omega_2 \) → \( P_{LR} = 2 \)

--

**Bandpass and Bandstop Transformations**

When \( \omega = \omega_o \)

\[
1 \left( \frac{\omega_o - \omega_o}{\omega_o - \omega_o} \right) = 0
\]

When \( \omega = \omega_1 \)

\[
1 \left( \frac{\omega_o, \omega_1}{\omega_o, \omega_1} \right) = 1 \left( \frac{\omega_1^2 - \omega_o^2}{\omega_o \omega_1} \right) = -1
\]

\[
\frac{\omega_o}{\omega_2 - \omega_1} \left( \frac{\omega_o - \omega_o}{\omega_o - \omega_o} \right) = \frac{\omega_o}{\omega_2 - \omega_1} \left( \frac{\omega_1^2 - \omega_o^2}{\omega_o \omega_1} \right) = \frac{1}{\omega_2 - \omega_1} \left( \frac{\omega_1 - \omega_o}{\omega_1 - \omega_o} \right)
\]

\[
\frac{1}{\omega_2 - \omega_1} \left( \frac{\omega_1 - \omega_1^2}{\omega_1 - \omega_1^2} \right) = \frac{\omega_1 - \omega_1}{\omega_1 - \omega_1} = -1
\]

When \( \omega = \omega_2 \)

\[
1 \left( \frac{\omega_o - \omega_o}{\omega_o - \omega_o} \right) = 1 \left( \frac{\omega_2^2 - \omega_o^2}{\omega_o \omega_2} \right) = 1
\]

\[
\frac{\omega_o}{\omega_2 - \omega_1} \left( \frac{\omega_o - \omega_o}{\omega_o - \omega_o} \right) = \frac{\omega_2 - \omega_1}{\omega_2 - \omega_1} = 1
\]

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Bandpass and Bandstop Transformations

Series inductance is transformed into

\[ L'_k = \frac{L_k}{\Delta \omega_o} \]
\[ C'_k = \frac{\Delta}{\omega_o L_k} \]

Shunt capacitance is transformed into

\[ L'_k = \frac{\Delta}{\omega_o C_k} \]
\[ C'_k = \frac{C_k}{\Delta \omega_o} \]

Bandstop

Series inductance is transformed into

\[ L'_k = \frac{\Delta L_k}{\omega_o} \]
\[ C'_k = \frac{1}{\omega_o \Delta L_k} \]

Shunt capacitance is transformed into

\[ L'_k = \frac{1}{\omega_o \Delta C_k} \]
\[ C'_k = \frac{\Delta C_k}{\omega_o} \]
Prototype Filter Transformation

\[
\Delta = \frac{\omega_2 - \omega_1}{\omega_o} \quad \omega_o = \sqrt{\omega_1 \omega_2}
\]

Example 8.4

Design a bandpass filter having a 0.5 dB equal-ripple, with \(N=3\). The center frequency is 1 GHz, the bandwidth is 10%, and the impedance is 50 Ω.

Answer:

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<th>(N)</th>
<th>(g_1)</th>
<th>(g_2)</th>
<th>(g_3)</th>
<th>(g_4)</th>
<th>(g_5)</th>
<th>(g_6)</th>
<th>(g_7)</th>
<th>(g_8)</th>
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</table>

Element Values for Equal-Ripple Low-pass filter prototypes
\((g_o=1, \omega_c=1, N=1 \text{ to } 10, \text{ 0.5 dB ripple})\)

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Example 8.4

\[ g_1 = 1.5963 = L_4 \]
\[ g_2 = 1.0967 = C_2 \]
\[ g_3 = 1.5963 = L_3 \]
\[ g_4 = 1.0000 = R_L \]

\[ \Delta = \frac{\omega_2 - \omega_1}{\omega_0} = 0.1 \]

\[ L_1' = \frac{L_1Z_o}{\omega_0 \Delta} = \frac{1.5963 \times 50}{2 \pi \times 10^9 \times 0.1} = 127.0 \text{ nH} \]
\[ L_2' = \frac{\Delta Z_o}{\omega_0 C_2} = 0.726 \text{ nH} \]

\[ C_1' = \frac{\Delta}{\omega_0 L_1 Z_o} = 0.199 \text{ pF} \]
\[ C_2' = \frac{C_2}{\omega_0 \Delta Z_o} = 34.91 \text{ pF} \]

\[ L_3' = \frac{L_3 Z_o}{\omega_0 \Delta} = 127.0 \text{ nH} \]

\[ C_3' = \frac{\Delta}{\omega_0 L_3 Z_o} = 0.199 \text{ pF} \]
Lecture # 10

- Filter Design By The Insertion Loss Method
  - Design Steps
    - Low-pass prototype design. (✓)
    - Scaling and conversion. (✓)
    - Implementation.
      - Using Stubs.
      - Using High-Low Impedance Sections.
    - Theory of periodic structures.

The process of filter design using IL method

Filter specifications → Low-pass prototype design → Scaling and conversion → Implementation
Filter Implementation Using Stubs

- Problems of filter design at microwave frequencies:
  - Lumped elements are generally available only for a limited range of values and difficult to implement at microwave frequencies.
  - Distance between filter components are not negligible.

- Solving Problem:
  - Convert lumped elements to transmission line sections. (Richard’s Transformation)
  - Separate filter elements using transmission line sections. (Kuroda’s Identities)

Richard’s Transformation

\[ \Omega = \tan \beta \ell = \tan \left( \frac{\omega \ell}{v_p} \right) \]

\[ jX_L = j\Omega L = jL \tan \beta \ell \]

\[ jB_C = j\Omega C = jC \tan \beta \ell \]

\[ Z_{in} = jZ_o \tan \beta \ell \]

\[ Z_{in} = -jZ_o \cot \beta \ell \]

Cutoff frequency occurs at \( \omega_c = 1 \).

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Kuroda’s Identities

• The four Kuroda’s Identities use redundant transmission line sections to achieve a more practical microwave filter implementation by performing any of the following operations:
  - Physically separate transmission line stubs.
  - Transform series stubs into shunt stubs, or vice versa.
  - Change impractical characteristic impedances into more realizable ones.

\[
\frac{\lambda}{8} = \frac{Z_1}{Z_2} = \frac{Z_2}{n^2} = \frac{1}{n^2 Z_2} = n^2 Z_1 = \frac{Z_1}{n^2}
\]

\[
n^2 = 1 + \frac{Z_2}{Z_1}
\]

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Kuroda’s Identities (Example)

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Kuroda's Identities (Example)

Let \( \tan \beta l = \Omega \)

\[ \sqrt{1+\Omega^2} \]

\[ \cos \beta l = \frac{1}{\sqrt{1+\Omega^2}} \]

\[
\begin{bmatrix}
A & B \\
C & D \\
\end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix}
1 & j\Omega Z_i \\
Z_1 & 1 \\
\end{bmatrix}
\]

Similarly

\[ \beta \quad Z_o = \frac{Z_2}{n^2} \]

\[
\begin{bmatrix}
A & B \\
C & D \\
\end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix}
1 & j\Omega Z_i \\
Z_1 & 1 \\
\end{bmatrix}
\]

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Example 8.5

Design a low-pass filter for fabrication using microstrip lines. The specifications are: cutoff frequency of 4 GHz, third order, impedance of 50 Ω, and a 3 dB equal ripple characteristic.

Answer

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<tr>
<th>N</th>
<th>g₁</th>
<th>g₂</th>
<th>g₃</th>
<th>g₄</th>
<th>g₅</th>
<th>g₆</th>
<th>g₇</th>
<th>g₈</th>
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Element Values for Equal-Ripple Low-pass filter prototypes (g₁₀=1, ωₙ=1, N=1 to 10, 3 dB ripple)

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**Example 8.5**

\[ g_1 = 3.3487 = L_1 \]
\[ g_2 = 0.7117 = C_2 \]
\[ g_3 = 3.3487 = L_3 \]
\[ g_4 = 1.0000 = R_L \]

Using Richard’s Transforms to convert inductors and capacitors to series and shunt stubs.

Unit elements at ends of filter matched to load; hence they have no effect on the circuit.

Second Kuroda identity

\[ n^2 = 1 + \frac{Z_2}{Z_1} = 1 + \frac{1}{3.3487} = 1.299 \]

\[ l = \lambda/8 \text{ at } \omega = 1 \]
Example 8.5

Impedance and frequency scaling

\[ Z_0 = 217.5 \, \Omega \quad l = \lambda/8 \text{ at } 4 \, \text{GHz} \]

\[ Z_0 = 64.9 \, \Omega \quad Z_0 = 70.3 \, \Omega \quad Z_0 = 64.9 \, \Omega \]

Microstrip Fabrication

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Example 8.5

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Another Solution

\[ g_1 = 3.3487 = C_1 \]
\[ g_2 = 0.7117 = L_2 \]
\[ g_3 = 3.3487 = C_3 \]

Using Richard’s Transforms to convert inductors and capacitors to series and shunt stubs.

Another Solution

\[ n^2 = 1 + \frac{0.2986}{1} = 1.2986 \]
Another Solution

\[ Z_o = 0.7117 \quad Z_o = 0.7701 \]

\[ Z_o = 0.2986 \]

Another Solution

\[ n^2 = 1 + \frac{0.2299}{0.7117} = 1.3230 \]
\[ n^2 = 1 + \frac{1}{0.7701} = 2.2986 \]
Another Solution

\[ Z_o = 0.2986 \quad Z_o = 0.9416 \quad Z_o = 2.2985 \]