IMPROVED CLOSED FORM SOLUTION FOR CENTER FED STRAIGHT WIRE ANTENNAS

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Abstract—The equivalent TEM mode theory, introduced by Fahmy and Eshrah in 2002[1], [2], [3], modeled the antenna as the inner conductor of a fictitious lossy transmission line with complex propagation constant and the proposed values of these constants are listed in tables without closed form expressions. The model parameters extraction procedure was rather complicated with some discrepancy in the input impedance values with respect to the results of the well established moment method solution. This paper uses the same model but the model parameters are extracted with a simple and systematic approach. The propagation constants are represented in closed form empirical expressions including only few terms yielding input impedance values in complete agreement with the well established moment method.

I. INTRODUCTION

The equivalent TEM mode theory introduced by Fahmy and Eshrah was based on a novel idea of modeling the radiated power of the antenna in the form of distributed losses along a fictitious lossy transmission line[1], [2], [3]. This idea has the advantages of the simplicity of the analytical approach[4], [5] and the accuracy of the numerical approach. It can describe the antenna parameters, namely the current distribution, the far field radiation and the input impedance in closed form expressions and the results obtained using this model are close to the numerical results. The equivalent TEM mode provides simple expressions for the antenna characteristics, which save the physical insight to the whole problem, without losing the accuracy. Modeling the antenna as a lossy transmission line results in describing the antenna current distribution, far field radiation and input impedance in terms of the fictitious transmission line parameters. These parameters are the characteristic impedance $Z_0$, the attenuation constant $\alpha$ and the phase constant $\beta$. On the other hand the previous model has some limitations: the procedure of extracting the model parameters is rather complicated and has no regular approach and the model is accompanied with long tables of different parameters.

This paper introduces a systematic procedure for extracting the model parameters. The model parameter extraction introduced in this paper is based on calculating the values of the phase constant and the attenuation constant of the transmission line to have exactly the same input impedance. Thus equating the input impedance of transmission line with the input impedance obtained from the method of moments results in having one complex equation in two unknowns $\alpha$ and $\beta$. These constants are then interpolated using appropriate empirical formulas. The next section presents the mathematical formulation of the expressions needed to extract the antenna parameters. Then the formulation results are presented in section III, where comparison between the results of the introduced model and the numerical technique results using AWAS is illustrated. Finally, section IV concludes the paper.

II. FORMULATION

The center fed antenna is modeled as the inner conductor of a fictitious lossy coaxial cable with open circuit terminations at its two extremities. The fictitious transmission line is slightly lossy carrying a TEM mode with the complex propagation constant $\gamma$ where the losses represent the radiated power from the antenna. The propagation factor is thus $e^{\pm j\gamma z} = e^{\pm j\alpha z} = e^{\pm j(\alpha + j\beta)z}$. The complex propagation constant $\gamma$ can be written in the following form,

$$\gamma = jk = \alpha + j\beta$$  \hspace{2cm} (1)

where $\alpha$ is the attenuation constant and $\beta$ is the phase constant.

A. Antenna Input Impedance

The voltage and the current on this transmission line can be related to each other using the well known telegraphists equations[6]. So the differential equations representing this relations can be written as:

$$\frac{dI(z)}{dz} = -Y V(z)$$  \hspace{2cm} (2)

$$\frac{dV(z)}{dz} = -ZI(z) + V_{in}\delta(z - z_f)$$  \hspace{2cm} (3)

where the $Y$ and $Z$ are, respectively, the shunt admittance and the series impedance per unit length of the transmission line[6]. Aside from the delta function in Equation 3, the equations represent the homogeneous telegraphists equation, where the delta function is the feeding excitation of the
antenna at \( z = 0 \). The line characteristic impedance \( Z_0 \) and the propagation constant \( \gamma \) are related to \( Y \) and \( Z \) through,

\[
Z_0 = \sqrt{\frac{Z}{Y}}, \quad \gamma = \sqrt{2ZY} \tag{4}
\]

By differentiating equation 2 with respect to \( z \) then using Equation. 3, the second order differential equation that describes the current wave along the transmission line can be written as,

\[
\frac{d^2I(z)}{dz^2} = \gamma^2 I(z) - Y V_{in} \delta(z), \tag{5}
\]

Solving the previous differential equation the current and voltage distributions will be obtained

\[
I(z) = \frac{V_{in}}{2Z_0 \cosh(\gamma l)} \sinh(\gamma(l - |z|)) \tag{6}
\]

\[
V(z) = \frac{V_{in}}{2 \cosh(\gamma l)} \cosh(\gamma(l - |z|)) \tag{7}
\]

Thus, the input impedance will take the following form

\[
Z_{in} = \frac{V_{in}}{I(0)} = -2Z_0 \coth(\gamma l) \tag{8}
\]

Assuming that the characteristics impedance can be approximately considered as a real value, the input resistance and reactance can be found as,

\[
R_{in} = Z_0 \frac{\sinh(2\alpha l)}{\cosh(2\alpha l) - \cos(2\beta l)} \tag{9}
\]

\[
X_{in} = -Z_0 \frac{\sin(2\beta l)}{\cosh(2\alpha l) - \cos(2\beta l)} \tag{10}
\]

**B. Far Field radiation**

To get the far field radiation, let the straight antenna wire be oriented along the \( z \)-axis, and assume that the antenna is sufficiently thin so that only axial current flow is considered. Accordingly, the magnetic vector potential will have only a \( z \)-component. An expression for the magnetic vector potential \( A \) is obtained using the following formula:

\[
A_z = \frac{\mu}{4\pi} \int e^{-j\kappa_0 R} \frac{I(z')}{R} dz', \tag{11}
\]

where \( k_0 \) is the value of the wave number in free space. The approximate equation that relates the radiated fields to the magnetic vector potential can be written as

\[
E_\theta = j\omega A_z \sin(\theta) = \frac{j\omega \mu}{4\pi} \frac{e^{-j\kappa_0 r}}{r} \sin(\theta) \int_{-l}^{l} e^{j\kappa_0 z' \cos(\theta)} I(z') dz' \tag{12}
\]

The previous integral is performed using the expression of the current distribution and the following equation is obtained

\[
E_\theta = \frac{j\omega \mu}{4\pi k_0 Z_0 \cos(\kappa l)} \frac{I_{in}}{k/k_0 - k/\cos(\theta)} \sin(\theta) \left[ (\cos(\kappa l \cos(\theta)) - \cos(\kappa l)) \right] \tag{13}
\]

**C. The Model Extraction Procedure**

The parameters of this model are the characteristic impedance \( Z_0 \), the attenuation constant \( \alpha \) and the phase constant \( \beta \). The parameters extraction procedure are:

1) *Estimation of the characteristic impedance \( Z_0 \):* It is required to estimate the radius of the fictitious outer conductor where the relation between the characteristic impedance and the outer conductor radius is

\[
Z_0 = \frac{\eta}{2\pi} \ln \left( \frac{b}{a} \right), \tag{14}
\]

where \( a \) is the radius of the inner conductor of the fictitious transmission line which is also the radius of the actual antenna, \( b \) is the radius of the outer conductor and \( \eta \) is the free space intrinsic impedance. The expressions of the input impedance obtained by the induced EMF method based on sinusoidal current distribution with maximum current \( I_m \) are expressible by Equations 8, 60 and 8.61 in Reference [7]. The input reactance of the induced EMF method is compared with the input reactance of the proposed model in equation 10 with the initial values of \( \beta \) and \( \alpha \), and by trying many values of \( Z_0 \) one selects the value of \( Z_0 \) that makes the value of \( X_{in} \) in both expressions agree with each other at odd multiples of \( \lambda/2 \) of the antenna length. It is clear that the value of \( X_{in} \) obtained using the induced EMF method deviates from the exact value around the singularities, so the comparison is performed in between singularities to use the most appropriate values obtained based on the induced EMF method.

The initial values of \( \alpha \) and \( \beta \) used in this comparison can be written as functions of the characteristic impedance in the following forms:

\[
\beta = \frac{2\pi}{\lambda} \quad \text{and} \quad \alpha = \frac{1}{2l} \ln \left( \frac{R_{rad}}{Z_0} \right) \tag{15}
\]

As the phase constant \( \beta \) initial value is assumed to be the free space phase constant and the attenuation constant initial value is obtained via equating the power loss in the model with the radiated power of the antenna where \( R_{rad} \) is obtained using the induced EMF method.

This estimation process leads to choosing the value of \( Z_0 \) in the following form

\[
Z_0 = \frac{\eta}{2\pi} \left( \frac{0.15l}{\alpha} \right) \quad \Rightarrow \quad b = 0.15l \tag{16}
\]

2) *Extracting Accurate Values for The Propagation Constant \( \gamma \):* This step is the major difference between the procedure used in the previous work[1], [2], [3] and the work done in this paper. It is clear from the above analysis that the target of the parameter estimation process is to have appropriate values for the attenuation constant, the phase constant and the characteristics impedance. Equations 15, are an initial representation for the propagation constant \( \gamma \). A more exact formulation for \( \gamma \) can be obtained from the expression for the input impedance given by equation 8.
The constants of the previous empirical equation are listed in table I for the case of symmetrically fed straight dipole. Where \( Z_{in} \) is obtained via substitution in equation 8.60 in

\[
Z_{in} = \frac{1}{1 - \frac{x}{Z_0}} = \frac{1}{1 - \frac{x}{Z_0}} \quad (17)
\]

where \( x = e^{-2j\beta l} \). Let

\[
A = \frac{Z_{in}}{2Z_0} = 1 + \frac{x}{1 - x} \quad (18)
\]

Notice that \( Z_0 \) is already given by equation 16 and \( Z_{in} \) is taken from the commercial software AWAS using the well established moment method. The above equation yields the value of \( x \) as:

\[
x = \frac{A - 1}{A + 1} = e^{-2j\beta l} = e^{-2\alpha l} e^{-2j\beta l} \quad (19)
\]

Now the propagation constants \( \alpha \) and \( \beta \) of the fictitious transmission line are related to \( x \) by

\[
\alpha = \frac{-1}{2l} \ln |x| \quad (20)
\]

\[
\beta = \frac{-1}{2l} \angle x \quad (21)
\]

III. RESULTS

The values of the attenuation and the phase constants determined from equations 19, 20 and 21 are solved and plotted versus \( L/\lambda \) in Figure 1.

These values are listed in tables in reference [8]. Each table consists of more than 600 points and to have these values in a compact form empirical formulas are introduced as an interpolating function that replaces these table. The suggested empirical formulas are:

\[
\beta l = a_{31} k_0 l - b_{31} \sin(c_{31} k_0 l) + d_{31} \quad (22)
\]

\[
\alpha l = 0.5a_{21} \sinh^{-1} \left( \frac{b_{21} R_{rad}}{Z_0} \right) \quad (23)
\]

Where \( R_{rad} \) is obtained via substitution in equation 8.60 in Reference [7] where \( k l = c_{31}/3l \)

The constants of the previous empirical equation are listed in table I for the case of symmetrically fed straight dipole.

Equations 22 and 23 are closed form empirical expressions for the propagation constants, to be immediately substituted in equations 9 and 10 yielding the input impedance of the center fed antenna depicted in Figure 2.

It is clear in this comparison that there is no difference between the two curves. That is very normal because the values of \( \alpha \) and \( \beta \) are chosen to have apriori the same input impedance. It is clear also that there is a very small percentage error in the curves based on the empirical formulas.

The current distribution will be obtained by substituting in equation 6 using the values of \( \alpha \) and \( \beta \) obtained before. The comparison between the current distribution in the proposed model and the current distribution obtained using the method of moment is shown in Figures 3 through 4.

It is shown that the current distribution of the proposed model is very close to the current distribution obtained by MoM, so it is expected to have almost the same far field radiation. Figure 5 shows a sample of the far field radiation Pattern for center fed antenna with different electrical lengths.

IV. CONCLUSION

This paper introduces a modified model to that presented in [3] for the center fed straight wire antenna with a straight forward, simple and systematic methodology for the model parameter extraction procedure. The proposed model gives closed form expressions for the antenna parameters of high accuracy compared to the numerically calculated results.

The input impedance calculated using the model is almost identical with the input impedance calculated numerically. The current distribution obtained using the model has a small error specially in the case of odd multiples integer of half wavelength dipoles. , and the error increases near multiple wavelengths dipole length. The error increased in this case because the assumed value of the characteristics impedance is more suitable for half wave length dipole and odd multiple integer of \( \lambda/2 \) so the max percentage error occurs between \( \lambda/2 \) and \( 3\lambda/2 \). Changing the value of the characteristic impedance affect the error in the current distribution.

Another important point is that, instead of the long tables of the attenuation constant and the phase constant we can use empirical expressions, which is valid over an acceptable range antenna dimensions.

REFERENCES


Fig. 1. The average attenuation constant and the phase constant for center fed antennas with different length to diameter ratios.

Fig. 2. The Input impedance for the straight wire antenna with length to diameter ratio of 400.

Fig. 3. The current distribution of a complete wave length dipole for different antenna length to diameter ratios.

Fig. 4. The current distribution of a one and a half wave length dipole for different antenna length to diameter ratios.

Fig. 5. The far field radiation of one and a half wavelength dipole for different length to diameter ratios.