COMM 907: Spread Spectrum Communications

Lecture 6

4. Detection Techniques of CDMA Systems
Detection Techniques of CDMA

Data detection

Single-user
- Conventional detector

Multiuser
- Decision driven
  - SIC
  - PIC
  - Decision Feedback
- Joint detection
  - MLSE
  - Decorrelator
  - MMSE
Detection Techniques of CDMA

• **A single-user detector** is defined as a receiver structure that requires no information regarding the other (interfering) users present in the system, and demodulates the data signal of *one user* only.

• **A multi-user detector** defined as a receiver structure that requires information regarding the other (interfering) users present in the system and demodulates the data signal of *all* the users.

- *In joint detection* the detector consists of (but not necessarily) a bank of matched filters followed by linear transformation (ex: Minimum Mean Sequence Estimator (MMSE) & Decorrelating detectors) or nonlinear transformations (ex: Maximum Likelihood Sequence Estimator (MLSE)) on the matched filter outputs.

- *Decision-driven* techniques characterized by the regeneration and subtraction of data estimates.
Synchronous and Asynchronous CDMA

- **Asynchronous CDMA**: In cellular CDMA systems, the up link (mobile to base station) is often not synchronized. The signal of each user arrives at a different propagation delay, given by $\tau_i \in [0, T_s), i = 1, 2, \ldots, K$. Therefore offsets are introduced to the model due to lack of the alignment of bit timing of K different users at the receiver.

- **Synchronous CDMA**: The signal of each user arrives at the same time. Synchronous CDMA is impractical to apply because each user transmit at any time.

Therefore, in wireless CDMA-systems as in IS-95, cdma2000, WCDMA and IEEE 802.11 users are asynchronous.
The basic CDMA receiver, popularly known as a conventional detector employs a single user matched filter (SUMF).

In SUMF detector, the signal from each user is demodulated individually without any knowledge of signals from other users as shown in Fig. 3.4.
Single User Matched Filter receiver (conventional Detector)

Fig. 3.4: Single user matched filter detector.
• Since the detection of a user is independent of the other users in the system, the number of computations needed for detection does not increase with the number of users.

Therefore, the time complexity per bit of the conventional detector is constant.
The received baseband signal from K-users assuming perfect power control in AWGN is given by

$$r(t) = \sum_{i=1}^{K} \sqrt{P} b_i(t) c_i(t) + n(t) \quad t \in [0, T_b]$$

(3.1)

where $T_b$ is the bit period, $b_i(t) \in [-1,1]$ is the bit transmitted by the $i$-th user, and $n(t)$ is white Gaussian noise with power spectral density of $N_0/2$ W/Hz. The signal $c_i(t)$ is the deterministic signature waveform (PN code waveform) assigned to the $i$-th user, normalized so as to have unit energy.

The output of the matched filter which contains the desired noise and other interfering users (MAI) can be expressed as:

$$y_i = \frac{1}{T_b} \int_{0}^{T_b} r(t) c_i(t) \, dt = \sqrt{P} b_i + \frac{1}{T_b} \sum_{j \neq i} \sqrt{P} b_j \rho_{ij} + n_i$$

Where

$$\rho_{ij} = \langle c_i, c_j \rangle = \int_{0}^{T_b} c_i(t) c_j(t) \, dt$$

is the cross correlation between the signature waveforms. The normalized cross correlation is

$$\rho_{ij} = \frac{1}{N} \sum_{n=1}^{N} c_i(n)c_j(n)$$

MAI term

$$\frac{1}{N} \sum_{n=1}^{N} c_i^2(n) = 1$$
• The bit decisions directly based on the sign of the matched filter outputs as follows:

\[ \hat{b}_i = \text{sgn}(y_i) \]

• The assumption of neglecting signals from other users sharing the channel is valid only when the spreading sequences of all users are orthogonal to each other and remain so when the signal is received at the base station after passing through the wireless propagation environment.

• Even if the spreading sequences are chosen to be orthogonal to each other to start with, the orthogonality is lost when the signals pass through multipath environment and arrive at the base station with random delays.

• However, the cross-correlations among spreading sequences lead to multiple access interference (MAI) which, in general, increases with the number of users.
SUMF Receiver:

- The MF or conventional detector is considered the simplest detector and considered as the optimum filter to recover the CDMA signals.

- The main drawback of MF receiver is that it treats the multiple access interference (MAI) as noise so as the number of users increase its performance degrades.
Performance of Asynchronous CDMA using SUMF

Assume that the desired user is user 1, and all other (K-1) users are interferers. Assume also \( \tau_i = 0 \), then the received signal, \( r(t) \), can be written as

\[
r(t) = \sum_{i=1}^{K} \sqrt{P} b_i(t - \tau_i) c_i(t - \tau_i) + n(t)
\]

The output of the matched filter for the first user \( y_1 = \frac{1}{T_b} \int_0^{T_b} r(t) c_1(t) \, dt \) can be expressed as

\[
y_1 = \frac{1}{T_b} \int_0^{T_b} \sum_{i=1}^{K} \sqrt{P} b_i(t - \tau_i) c_i(t - \tau_i) c_1(t) \, dt + \frac{1}{T_b} \int_0^{T_b} n(t) c_1(t) \, dt
\]

\[
= \frac{1}{T_b} \int_0^{T_b} \sqrt{P} b_1(t) c_1^2(t) \, dt + \frac{1}{T_b} \sum_{i=2}^{K} \int_0^{T_b} \sqrt{P} b_i(t - \tau_i) c_i(t - \tau_i) c_1(t) \, dt + \frac{1}{T_b} \int_0^{T_b} n(t) c_1(t) \, dt
\]

Assume the first bit \( b_1(t) = 1 \), then

\[
y_1 = \sqrt{p} + \sum_{i=2}^{K} I_i + n_1
\]

(3.7)

Since \( n(t) \) is the zero-mean AWGN, with variance \( N_0 / 2 \)
then the third term of (3.7) is a Gaussian variable with zero mean and variance \( N_0 / 2T_b \).

The second term in the last line of (3.7) is the MAI component from all other \((K-1)\) users, which from the central limit theorem, the summation of \((K-1)\) independent random variables can be modeled as a Gaussian distribution, and hence in our analysis, the MAI is approximated by a Gaussian random variable. The MAI has a zero mean and variance:

\[
\sigma^2 = \frac{(K - 1) P}{3N}
\]

See reference book for proof.
Since the noise and the MAI are Gaussian, the bit error probability of the asynchronous CDMA system is given by

\[ P_e = Q\left( \frac{\mu_{y_1}}{\sigma_{y_2}} \right) \]  \hspace{1cm} (3.9)

where \( \mu_{y_1} \) and \( \sigma_{y_i}^2 \) are respectively the mean and the variance of \( y_1 \) (given by (3.7)).

Assuming \( b_1 = 1 \), then using (3.7), \( \mu_{y_1} \) is given by

\[ \mu_{y_1} = \sqrt{P} \]  \hspace{1cm} (3.10)

Since the noise and the MAI are independent, the variance of their sum is the sum of their variances, then \( \sigma_{y_1}^2 \) is given by

\[ \sigma_{y_1}^2 = \frac{N_0}{2T_b} + \frac{(K - 1) P}{3N} \]  \hspace{1cm} (3.11)
Then, by substitution of (3.10) and (3.11) in (3.9) we have
Using the relation $E_b = PT_b$, (3.12) becomes

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0 + \frac{2E_b(K-1)}{3N}}}\right)$$

$$= Q\left(\left[\frac{N_0}{2E_b} + \frac{K-1}{3N}\right]^{-1}\right)$$

(3.13)

Where $N$ is the code length
Fig. 3.5 Theoretical performance of asynchronous CDMA system over Gaussian channel: effect of sequence length. The number of users is fixed at $K=5$, different sequence lengths, $N$, are used.
Performance of Asynchronous CDMA using SUMF in Presence of interference

- Assume perfect power control, then the received baseband signal from K-users in presence of jamming is given by:

\[
    r(t) = \sum_{i=1}^{K} \sqrt{P} b_i(t - \tau_i)c_i(t - \tau_i) + n(t) + J(t) \quad t \in [0, T_b]
\]

- The performance of the system in presence of partial band interference is performed by adding a white Gaussian noise with a fraction \(\delta\) of the total bandwidth. Therefore the bit error probability becomes:

\[
P_e = Q\left(\sqrt{\frac{2E_b}{N_0} + \frac{2E_b(K-1)}{3N} + \frac{N_j}{\delta}}\right)^{-1} = Q\left(\sqrt{\frac{N_0}{2E_b} + \frac{K-1}{3N} + \frac{N_j}{2E_b\delta}}\right)^{-1}
\]

where \(\delta = W_j / W_{ss}\)
• In broadband jamming, the jammer waveform is Gaussian noise that is spectrally white over the system bandwidth. By substituting in the previous equation, the bit error probability in presence of broadband jamming is given by

\[ P_e = Q \left( \sqrt{\frac{N_0}{2E_b}} + \frac{K-1}{3N} + \frac{N_j}{2E_b} \right)^{-1} \]

The probability of error of CDMA system in presence of broadband and partial band jamming (with \( \sigma = 0.25, 0.5, 0.75, 1 \)) is shown in Fig. 3.7.
The figure shows that as the value of $\delta$ increases the effect of jamming decreases. This is because as $\delta$ increases, the jamming bandwidth increases and then the jamming power decreases.

Fig. 3.7. Comparison of probability of error for CDMA systems under different partial band jamming factor $\delta$ with number of users = 5, code length =127, SJR = 10 dB.

The figure shows that as the value of $\delta$ increases the effect of jamming decreases. This is because as $\delta$ increases, the jamming bandwidth increases and then the jamming power decreases.
The figure shows that the partial band jamming causes an error floor in the performance of the CDMA system. It is also shown that the performance of the system degrades as the number of users increases. This is due to the increase of MAI.
Capacity of CDMA system

It can be shown that, for a given probability of error determined by the value for \( \frac{E_b}{N_0} \), the total number of simultaneous active users who can have access to the system is given by:

\[
K \leq 1 + N \left( \frac{1}{\frac{E_b}{\hat{N}_0}} - \frac{1}{\frac{E_b}{N_0}} \right)
\]

where \( N \) is the processing gain of the DS-spread spectrum system assuming \( T = NT_c \).

\( \hat{N}_0 \) is the total noise density which is given by sum of a Gaussian noise density \( (N_0) \) and interference power density \( (I_0) \).
(2) Decorrelator Detector

- The decorrelating detector is shown in Fig. 4.3. It consists of a bank of matched filters followed by a linear transformation that multiplies the output of the matched filter bank with the inverse of the correlation matrix.

- This removes all the interference for a user $i$, caused by any of the other users. The only source of interference left is the enhanced background noise.
Decorrelator Detector

Fig. 4.3: Decorrelator detector.
The received signal:

\[
r(t) = \sum_{i=1}^{K} \sqrt{P} b_i(t) c_i(t) + n(t) \quad t \in [0,T_b]
\]

we can express the output of the \( i-th \) matched filter as

\[
y_i = \frac{1}{T_b} \int_{0}^{T_b} r(t) c_i(t) \, dt = \sqrt{P} b_i + \sum_{j \neq i} \sqrt{P} b_j \rho_{ji} + n_i
\]

where

\[
n_i = \frac{1}{T_b} \int_{0}^{T_b} n(t) c_i(t) \, dt \quad (4.4)
\]

We can express \( y_i \) in vector/matrix form as:

\[
y = RA\bar{b} + n \quad (4.5)
\]
\[ y_j = \frac{1}{T_b} \int_0^{T_b} \sum_{i=1}^{K} \sqrt{p_i} b_i(t) c_i(t) c_j(t) \, dt + \frac{1}{T_b} \int_0^{T_b} n(t) c_j(t) \, dt \]

\[ = \frac{1}{T_b} \sum_{i=1}^{K} \sqrt{p_i} b_i \rho_{ij} + n_j \text{ where } \rho_{ij} = \int_0^{T_b} c_i(t) c_j(t) \, dt \text{ and } j = 1 : K \]

\[ y_j = \frac{\sqrt{p_1}}{T_b} b_1 \rho_{1j} + \frac{\sqrt{p_2}}{T_b} b_2 \rho_{2j} + \ldots + \frac{\sqrt{p_K}}{T_b} b_K \rho_{Kj} + n_j \]

Then

\[ y_1 = \frac{\sqrt{p_1}}{T_b} b_1 \rho_{11} + \frac{\sqrt{p_2}}{T_b} b_2 \rho_{21} + \ldots + \frac{\sqrt{p_K}}{T_b} b_K \rho_{K1} + n_1 \]

\[ y_2 = \frac{\sqrt{p_1}}{T_b} b_1 \rho_{12} + \frac{\sqrt{p_2}}{T_b} b_2 \rho_{22} + \ldots + \frac{\sqrt{p_K}}{T_b} b_K \rho_{K2} + n_2 \]

\[ y_K = \frac{\sqrt{p_1}}{T_b} b_1 \rho_{1K} + \frac{\sqrt{p_2}}{T_b} b_2 \rho_{2K} + \ldots + \frac{\sqrt{p_K}}{T_b} b_K \rho_{KK} + n_K \]
Let \( R = \begin{pmatrix} \rho_{11} & \rho_{21} & \rho_{K1} \\ \rho_{12} & \rho_{22} & \rho_{K2} \\ \vdots & \vdots & \vdots \\ \rho_{1K} & \rho_{2K} & \rho_{KK} \end{pmatrix}_{K \times K} \) represents the correlation matrix,

\[
A = \begin{pmatrix} \frac{\sqrt{p_1}}{T_b} & 0 & \ldots & \ldots & 0 \\ 0 & \frac{\sqrt{p_2}}{T_b} & \ldots & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & \frac{\sqrt{p_K}}{T_b} & 0 \end{pmatrix}_{K \times K}
\]
represents the amplitudes, \( b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_K \end{pmatrix} \), \( y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_K \end{pmatrix} \), and

\[
\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ \vdots \\ n_K \end{pmatrix}
\]

Then \( y = RAb + \mathbf{n} \)
Decorrelator Detector

\[ y = RAb + n \]  \hspace{1cm} (4.5)

Now let us simply multiply both side of (4.5) with the inverse of R (denoted by \( R^{-1} \)) assuming \( R^{-1} \) exists that is \( R \) is a square matrix, then

\[ R^{-1}y = Ab + R^{-1}n \]
Decorrelator Detector

Since $\mathbf{R}^{-1}\mathbf{R} = \mathbf{I}$ where $\mathbf{I}$ is the identity matrix and $\mathbf{IAb} = \mathbf{Ab}$, if $\sigma = 0$, we can simply take the components in (4.7) to recover the transmitted data:

$$\hat{b}_i = \text{sgn}((\mathbf{R}^{-1}\mathbf{y})_i)$$
$$= \text{sgn}([\mathbf{Ab}])_i$$
$$= b_i$$  \hspace{1cm} (4.8)

Let us bring in the noise now. Processing the matched filter bank outputs (4.7) with $\mathbf{R}^{-1}$ results in

$$\mathbf{R}^{-1}\mathbf{y} = \mathbf{Ab} + \mathbf{R}^{-1}\mathbf{n}$$  \hspace{1cm} (4.9)

Thus, (4.9) shows that the decorrelator detector completely eliminates the MAI at the expense of noise enhancement at the output of decorrelator detector. The disadvantage of the decorrelator detector is that it requires computation of the inverse of the matrix $\mathbf{R}$ which is difficult to perform in real time, specially for asynchronous CDMA.