Lecture 4: Analog Filters (continued)

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Active Filter: Low-Pass Filter

Sallen Key LPF

\[ K = 1 + \frac{R_b}{R_a} \]

\[
H(s) = \frac{V_o}{V_i} = \frac{K}{S^2 + \left( \frac{1}{R_2C_1} + \frac{1}{R_1C_1} + \frac{1-K}{R_2C_2} \right)S + \frac{1}{R_1R_2C_1C_2}}
\]
Active Filter: Low-Pass Filter

Sallen Key LPF

Using the standard notation:

\[ H_{LPF}(s) = \frac{G}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2} \]

\[ \omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \]

\[ Q = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \left( \frac{1}{R_2 C_1} + \frac{1}{R_1 C_1} + \frac{1 - k}{R_2 C_2} \right) \]
If no restriction is imposed on the gain $K$, we have five element values to design LPF circuit. Hence, we are free to make some arbitrary choices as follows:

**Design 1:** equal element values
In this design, we let $C_1 = C_2 = 1\, \text{F}$ and $R_1 = R_2 = R$.

**Design 2:** equal capacitance and equal feedback resistances
In this design, we choose $C_1 = C_2 = 1\, \text{F}$ and $R_a = R_b = R$.

**Design 3:** moderate-sensitivity design
In this design, we choose $C_1 = \sqrt{3} \, Q$ and $C_2 = 1\, \text{F}$ and $K = 4/3$.

**Design 4:** minimum sensitivity
In this design, we choose $K = 1$ and $R_1 = R_2 = 1\, \Omega$. 

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Active Filter: Low-Pass Filter

Definition of sensitivity

The sensitivity of some performance measure $y$ with respect to a network element value $x$ is defined by:

$$ S^y_x = \frac{x \ dy}{y \ dx} $$

If $y$ is a function of several variables $[y=f(x_1, x_2, \ldots, x_n)]$, then the sensitivity of $y$ with respect to $x_i$ is:

$$ S^y_{x_i} = \frac{x_i \ dy}{y \ dx_i} $$

Notes:

$$ S^y_{x_1} + S^y_{x_2} = S^y_{x_1/y_2} + S^y_{x_2} $$

$$ S^y_{x_1/y_2} = S^y_{x_1} - S^y_{x_2} $$
Example:
Design a fourth-order butterworth low-pass filter using the cascade of two Sallen-key biquads using Design 1 and 2 respectively. Then let \( w_0 = 2\pi \times 1000 \) rad/sec and use 0.1 \( \mu \)F capacitors. The normalized network function of the two low-pass filters are:

\[
\begin{align*}
H_1(s) &= \frac{G_1}{s^2 + 0.765367} \frac{1}{s + 1} \\
H_2(s) &= \frac{G_2}{s^2 + 1.847759} \frac{1}{s + 1}
\end{align*}
\]
Active Filter: High-Pass Filter

First order HPF (using Inverting op-amp configuration)

\[ H(s) = -\frac{R_2}{R_1 + \frac{1}{SC}} \]

\[ H(s) = -K \frac{S}{S + w_c} \]

\[ K = \frac{R_2}{R_1} \]

\[ w_c = \frac{1}{R_1 C} \]
Active Filter: High-Pass Filter

First order HPF (using Non-Inverting amplifier)

$$H(s) = \frac{(1 + \frac{R_3}{R_2})S}{S + \frac{1}{R_1 C_1}}$$

$$H(s) = K \frac{S}{S + w_c}$$

$$K = 1 + \frac{R_3}{R_2}$$

$$w_c = \frac{1}{R_1 C_1}$$
Active Filter: High-Pass Filter

Sallen Key HPF

\[ K = 1 + \frac{R_b}{R_a} \]

\[ H(s) = \frac{V_o}{V_i} = \frac{KS^2}{S^2 + \left(\frac{1}{R_2C_1} + \frac{1}{R_2C_2} + \frac{1}{R_1C_1} - K\right)S + \frac{1}{R_1R_2C_1C_2}} \]
Active Filter: High-Pass Filter

Sallen Key HPF

Using the standard notation:

\[ H_{HPF}(s) = \frac{GS^2}{S^2 + \left(\frac{w_o}{Q}\right)S + w_o^2} \]

\[ w_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \]

\[ Q = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} + \frac{1}{R_2 C_2} + \frac{1}{R_1 C_1} + \frac{1 - k}{R_2 C_1} \]
Active Filter: Band-Pass Filter

Sallen Key BPF

\[ K = 1 + \frac{R_b}{R_a} \]

\[
H(s) = \frac{V_o}{V_i} = \frac{K}{R_1 C_1} \frac{s}{s^2 + \left( \frac{1}{R_1 C_1} + \frac{1}{R_3 C_2} \right) s + \left( \frac{1}{R_1 C_1} + \frac{1}{R_3 C_1} + \frac{1 - K}{R_2 C_1} \right) s + \frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}}
\]
Active Filter: Band-Pass Filter

Sallen Key BPF

Using the standard notation:

\[
H_{BPF}(s) = \frac{G \left( \frac{w_o}{Q} \right) S}{S^2 + \left( \frac{w_o}{Q} \right) S + w_o^2}
\]

\[
w_o = \sqrt{\frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}}
\]

\[
Q = \frac{\sqrt{\frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}}}{1 + \frac{1}{R_1 C_1} + \frac{1}{R_3 C_2} + \frac{1}{R_3 C_1} + \frac{1-k}{R_2 C_1}}
\]
Active Filter: Band-rejection Filter

Sallen Key Band-Rejection Filter

\[ K = 1 + \frac{R_b}{R_a} \]

\[ H(s) = \frac{V_o}{V_i} = \frac{K \left( S^2 + \frac{1}{R^2 C^2} \right)}{S^2 + \left( \frac{4 - 2K}{RC} \right)S + \frac{1}{R^2 C^2}} \]
Active Filter: Band-rejection Filter

Sallen Key Band-Rejection Filter

Using the standard notation:

\[ H_{BRF}(s) = \frac{K (S^2 + w_o^2)}{S^2 + (\frac{w_o}{Q})S + w_o^2} \]

\[ w_o = \frac{1}{RC} \]

\[ Q = \frac{1}{4 - 2K} \]
Active Filter: Multiple-Feedback Biquads (MFB)

- The MFB topology is commonly used in filters that have high Q and require a high gain.
Active Filter: Multiple-Feedback Biquads (MFB)

Multiple Feedback Low-pass Biquad

\[ H(s) = \frac{V_o}{V_i} = -\frac{1}{s^2 + \frac{1}{C_1} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) s + \frac{1}{R_2 R_3 C_1 C_2}} \]

\[ H_{LPF}(s) = \frac{G}{s^2 + \left( \frac{w_o}{Q} \right) s + w_o^2} \]

\[ w_o = \frac{1}{\sqrt{R_2 R_3 C_1 C_2}} \]

\[ Q = \sqrt{\frac{C_1}{C_2}} \sqrt{\frac{R_2 R_3}{R_1}} + \sqrt{\frac{R_3}{R_2} + \frac{R_2}{R_3}} \]
Active Filter: Multiple-Feedback Biquads (MFB)

Multiple Feedback High-pass Biquad

\[ H(s) = \frac{V_o}{V_i} = -\frac{C_1 S^2}{C_2 S^2 + \frac{C_1 + C_2 + C_3}{R_2 C_2 C_3} S + \frac{1}{R_1 R_2 C_2 C_3}} \]

\[ w_o = \frac{1}{\sqrt{R_1 R_2 C_2 C_3}} \]

\[ \frac{w_o}{Q} = \frac{C_1 + C_2 + C_3}{R_2 C_2 C_3} \]
Active Filter: Multiple-Feedback Biquads (MFB)

Multiple Feedback bandpass Biquad

\[ H(s) = \frac{V_o}{V_i} = -\frac{1}{R_1 C_2} \frac{S}{S^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2}\right)S + \left[\frac{1}{R_1} + \frac{1}{R_3}\right]} \]

\[ w_o = \sqrt{\frac{1}{R_1} + \frac{1}{R_2 C_1 C_2}} \]

\[ \frac{w_o}{Q} = \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} \]

\[ |H(jw_o)| = \frac{R_2 / R_1}{1 + C_2 / C_1} \]
Active Filter: Multiple-Feedback Biquads (MFB)

Multiple Feedback band-rejection Biquad

\[
H(s) = \frac{V_o}{V_i} = -\frac{K \left( S^2 + \frac{1}{R^2 C^2} \right)}{S^2 + \frac{4 - 2K}{RC} S + \frac{1}{R^2 C^2}}
\]

\[
w_o = \frac{1}{RC}
\]

\[
Q = \frac{1}{4 - 2K}
\]

\[
K = 1 + \frac{R_b}{R_a}
\]
Active Filter: Multiple-Amplifier Biquads

The step from single-amplifier to multiple-amplifier biquadratic filter sections provides several benefits. The most important benefits are as follows:
- Reduced passive element spread, i.e., the ratio between the largest and smallest values of resistors and/or capacitors can be reduced compared to the single-amplifier case
- Multiple-amplifier biquads often provide lower sensitivities to both passive and active component
- Most of the filter parameters can be tuned independently.
- Designing multiple-amplifier filters, often the values of capacitors can be chosen freely. The filter parameters are then determined by resistors, which is less costly than by capacitors.

**Note:**
- On the other hand, these benefits must be paid for by increased space requirements and increased power dissipation. However, today there are low-cost and low-power integrated-circuit op-amps available with up to four op-amps on one chip. Therefore, size and power dissipation are often no longer the main problem.
- There are two very famous multiple amplifier biquads;
  1. Tow-Thomas Filter (TT),
  2. Kerwin- Huelsman- Newcomb Filter (KHN)
Multiple-Amplifier Biquads: Tow-Thomas Filter

By the direct analysis, three outputs are available:
- inverting lowpass output \((-V_{LP})\)
- noninverting lowpass output \((V_{LP})\)
- inverting bandpass output \((V_{BP})\)

\[
\frac{V_{LP}}{V_{in}} = \frac{1}{S^2 + \left(\frac{1}{R_1C_1}\right)S + \frac{1}{R_2R_3C_1C_2}}
\]

\[
w_o = \frac{1}{\sqrt{R_2R_3C_1C_2}}
\]

\[
Q = R_1 \sqrt{\frac{C_1/C_2}{R_2R_3}}
\]

\[
|H_{LP}(0)| = \frac{R_3}{R_4}
\]

\[
|H_{BP}(jw_o)| = \frac{R_1}{R_4}
\]
Multiple-Amplifier Biquads: Tow-Thomas Filter

- The Tow-Thomas Filter has the following advantages:
  - Its Q can be adjusted by varying a single resistance → R1 without disturbing $w_0$
  - The gain can be varied by varying a single resistance → R4 without disturbing $w_0$ and Q
  - The lowpass output can be either inverting or noninverting
  - The sensitivity of the Tow-Thomas biquad are generally comparable to those of the KHN biquad
Multiple-Amplifier Biquads: Kerwin- Huesman- Newcomb Filter

By the direct analysis, three outputs are available:

- noninverting High-pass output \( V_{HP} \)
- inverting band-pass output \(-V_{BP}\)
- noninverting low-pass output \( V_{LP} \)

\[
V_{LP} = \frac{V_{in}}{S^2 + \left[ \left( \frac{R_3}{R_5} \right) \left( \frac{R_5 + R_6}{R_3 + R_4} \right) \left( \frac{1}{R_1 C_1} \right) \right] S + \left( \frac{R_6}{R_5} \right) \frac{1}{R_1 R_2 C_1 C_2}}
\]

\[
V_{BP} = -\frac{V_{in}}{S^2 + \left[ \left( \frac{R_3}{R_5} \right) \left( \frac{R_5 + R_6}{R_3 + R_4} \right) \left( \frac{1}{R_1 C_1} \right) \right] S + \left( \frac{R_6}{R_5} \right) \frac{1}{R_1 R_2 C_1 C_2}}
\]

\[
V_{HP} = \frac{V_{in}}{S^2 + \left[ \left( \frac{R_3}{R_5} \right) \left( \frac{R_5 + R_6}{R_3 + R_4} \right) \left( \frac{1}{R_1 C_1} \right) \right] S + \left( \frac{R_6}{R_5} \right) \frac{1}{R_1 R_2 C_1 C_2}}
\]

Notes:

\[
V_{LP} = -\frac{1}{SR_2 C_2} V_{BP}
\]

\[
V_{BP} = -\frac{1}{SR_1 C_1} V_{HP}
\]

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The general transfer function of the second order active filter in the s-plane has the following form:

\[ H(s) = \frac{K_1 S^2 + K_2 S + K_3}{S^2 + \left(\frac{w_o}{Q}\right) S + w_o^2} \]

- \( K_1 = K_2 = 0 \) \( \rightarrow \) Low-Pass
- \( K_1 = K_3 = 0 \) \( \rightarrow \) Band-Pass
- \( K_2 = K_3 = 0 \) \( \rightarrow \) High-Pass
- \( K_1 = 1, K_2 = 0, \) and \( K_3 = w_o^2 \) \( \rightarrow \) band-reject
- \( K_1 = 1, K_2 = -\frac{w_o}{Q}, \) and \( K_3 = w_o^2 \) \( \rightarrow \) All-Pass