COMM 601: Modulation I

Lecture 3

Generation & Detection of Amplitude Modulation
Instantaneous Amplitude, Phase, and Frequency

\[ s(t) = \text{Re}\left\{ C(t) e^{j2\pi f_c t} \right\} \]

**Instantaneous Amplitude**

\[ a(t) = |C(t)| \]  \rightarrow \text{Magnitude of } C(t)

**Instantaneous Phase**

\[ \phi(t) = \text{Arg}(C(t)) \]  \rightarrow \text{Angle of } C(t)

**Instantaneous Frequency**

\[ f(t) = \frac{1}{2\pi} \frac{d}{dt} \{ \phi(t) \} \]
Instantaneous Amplitude, Phase, and Frequency of Conventional AM

\[ s(t) = A_c \left[ 1 + k_a m(t) \right] \cos(2\pi f_c t) \]

\[ s(t) = \text{Re}\left\{ C(t) e^{j2\pi f_c t} \right\} \]

\[ s(t) = \text{Re}\left\{ A_c \left[ 1 + k_a m(t) \right] e^{j2\pi f_c t} \right\} \]

\( C(t) = A_c \left[ 1 + k_a m(t) \right] \)

\( a(t) = |A_c \left[ 1 + k_a m(t) \right]| \)

\( \phi(t) = \text{Re}\left\{ A_c \left[ 1 + k_a m(t) \right] \right\} \)

\( f(t) = \frac{1}{2\pi} \frac{d}{dt} \{\phi(t)\} \)
Instantaneous Amplitude, Phase, and Frequency of Conventional AM (Single Tone Modulating Signal)

\[ s(t) = A_c \left[ 1 + k_a a_m \cos(2\pi f_m t) \right] \cos(2\pi f_c t) \]

\[ s(t) = \text{Re}\left\{ C(t) e^{j2\pi f_c t} \right\} \]

\[ s(t) = \text{Re}\left\{ A_c \left[ 1 + k_a a_m \cos(2\pi f_m t) \right] e^{j2\pi f_c t} \right\} \]

\[ C(t) = A_c \left[ 1 + k_a a_m \cos(2\pi f_m t) \right] \]

\[ a(t) = A_c \left[ 1 + k_a a_m \cos(2\pi f_m t) \right] \]
\[ = A_c \left[ 1 + k_a a_m \cos(2\pi f_m t) \right] \]

\[ \phi(t) = \text{Re}(C(t)) = 0 \]

\[ f(t) = \frac{1}{2\pi} \frac{d}{dt} \{ \phi(t) \} = 0 \]
Instantaneous Amplitude

Instantaneous Phase

Time [Sec.]
Generation of Conventional AM Signal

(1) Using Product Modulator

\[ s(t) = a_1 m(t) \cos(2\pi f_c t) + A_c \cos(2\pi f_c t) \]

\[ = A_c \left[ 1 + \frac{a_1}{A_c} m(t) \right] \cos(2\pi f_c t) \]
Generation of Conventional AM Signal

(2) Square Law Modulator

Let $v_i(t)$ be the input signal. The output of the nonlinear device can be expressed as

$$v_o(t) = \sum_{n=1}^{\infty} a_n v_i^n(t)$$

where $a_n$ are constants.
Suppose that the nonlinear device is approximated by a second order polynomial.

\[ v_o(t) = \sum_{n=1}^{2} a_n v_i^n(t) = a_1 v_i(t) + a_2 v_i^2(t) \]

Input to the nonlinear device

\[ v_i(t) = m(t) + A_c \cos(2\pi f_c t) \]
(2) Square Law Modulator (Cont.)

- Output of the nonlinear device

\[ v_o(t) = a_1[m(t) + A_c \cos(2\pi f_c t)] + a_2[m(t) + A_c \cos(2\pi f_c t)]^2 \]

\[ = a_1m(t) + a_1A_c \cos(2\pi f_c t) + a_2m^2(t) + a_2A_c^2 \cos^2(2\pi f_c t) + 2a_2A_cm(t) \cos(2\pi f_c t) \]

\[ = a_1m(t) + a_2m^2(t) + a_2A_c^2 \cos^2(2\pi f_c t) + A_c a_1 \left[ 1 + \frac{2a_2}{a_1} m(t) \right] \cos(2\pi f_c t) \]

\[ = a_1m(t) + a_2m^2(t) + \frac{1}{2} a_2A_c^2 [1 + \cos(4\pi f_c t)] + A_c a_1 \left[ 1 + \frac{2a_2}{a_1} m(t) \right] \cos(2\pi f_c t) \]

- The band pass filter with bandwidth 2\( W \) centered at \( f = f_c \) yields

\[ s(t) = A_c a_1 \left[ 1 + \frac{2a_2}{a_1} m(t) \right] \cos(2\pi f_c t) \]

where by design

\[ \frac{2a_2}{a_1} |m(t)| < 1 \]
Generation of Conventional AM Signal

(3) Switching Modulator:

We assume that the diode acts as an ideal switch:
- The diode will turn on When the carrier voltage \( c(t) > 0 \)
- The diode will turn off When the carrier voltage \( c(t) \leq 0 \)

Assume that \( A_c \gg m(t) \)

i.e the carrier dominates the behavior

Figure 3.17 Switching modulator and periodic switching signal.
• Let \( c(t) = A_c \cos(2\pi f_c t) \)

• The diode will turn on if \( c(t) > 0 \) and will turn off if \( c(t) \leq 0 \)

• The output across the load resistor is

\[
v_0(t) = \begin{cases} 
    v_i(t) & c(t) > 0 \\
    0 & c(t) \leq 0 
\end{cases}
\]

\[= v_i(t) g(t)\]

\[= [m(t) + A_c \cos(2\pi f_c t)] g(t)\]

• Since \( g(t) \) is a periodic rectangular function, the Fourier series is

\[
g(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t(2n-1)]
\]
• Hence

\[ v_0(t) = [m(t) + A_c \cos(2\pi f_c t)]g(t) \]

\[ = [m(t) + A_c \cos(2\pi f_c t)] \left( \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t(2n-1)] \right) \]

\[ = \frac{m(t)}{2} + \frac{A_c}{2} \cos(2\pi f_c t) + \frac{2}{\pi} m(t) \cos(2\pi f_c t) + \frac{2}{\pi} A_c \cos^2(2\pi f_c t) + \ldots \text{other high frequency terms} \]

\[ = \frac{A_c}{2} \left[ 1 + \frac{4}{\pi A_c} m(t) \right] \cos(2\pi f_c t) + \frac{m(t)}{2} + \frac{1}{\pi} A_c [1 + \cos(4\pi f_c t)] + \ldots \text{other high frequency terms} \]

- Passing \( v_0(t) \) through a bandpass filter centered around \( f_c \) and has BW=2W, we have

\[ s(t) = \frac{A_c}{2} \left[ 1 + \frac{4}{\pi A_c} m(t) \right] \cos(2\pi f_c t) \]

- BPF centered Around Carrier frequency with BW=2W
Demodulation of Amplitude Modulated signals

- **Demodulation**
  - The process of extracting the message signal from modulated signal

- **Type of demodulation**
  - **Coherent demodulation**
    - Local oscillator with same frequency and phase of the carrier at the receiver
    - DSB, SSB
  - **Noncoherent demodulation**
    - Envelope detector which does not require same frequency and phase of carrier
    - Easy to implement with low cost: Conventional AM
Envelope Demodulation:

- The block diagram of the conventional AM receiver is shown in fig. The LPF is used to make smoothing of the waveform after the envelope detector.
Envelope detector is a circuit that produces a waveform proportional to the envelope of the input signal.
Demodulation of Conventional AM
Envelope Detector

Remember:

- Short Time constant → Fast charging or discharging
- Long Time constant → Slow charging or discharging
Demodulation of Conventional AM
Envelope Detector

Generally we can judge from the second figure (by intuition) what are the conditions required for the system. Clearly, the period of $m(t)$ should be much larger than $c(t)$, meaning the inverse for the frequency. Another thing, the carrier amplitude $A_c$ should be large (to guarantee proper operation of the discharge).
Demodulation of Conventional AM-Envelope Detector

In the positive half cycle the diode is forward biased and the capacitor charges.

This continues till it reaches the peak value of the signal.
Demodulation of Conventional AM-Envelope Detector

Our first condition will be on the charging time: the charging time constant $R_s C$ ...

It should be shorter than the carrier period (check the graph).

so that the capacitor $C$ charges rapidly and thereby follows the applied voltage up to the positive peak when the diode is conducting. Otherwise, it is possible the charging will not reach the peak.

$$R_s C << \frac{1}{f_c}$$ (1)

If this was the charging curve:
- Too slow
- Does not reach the peak

If the time constant is long
Demodulation of Conventional AM-Envelope Detector

After falling below the peak the capacitor discharges through $R_l$. 

![Diagram of a diode circuit with $R_s$, $C$, and $R_l$ components, showing the output signal process.]
Demodulation of Conventional AM-Envelope Detector

The discharging process is governed by the discharging time constant $R_l C$.

The discharging time constant $R_l C$ must be long enough to ensure that the capacitor discharges slowly through the load resistor $R_l$ between positive peaks of the carrier wave, but not so long that the capacitor voltage will not discharge at the maximum rate of change of the modulating wave, that is

$$\frac{1}{f_c} \ll R_l C \ll \frac{1}{W}$$

(2)

Slow discharging

If this was the discharging curve: Too slow
Does not catch the changes in the message
# AM Broadcasting Station Standard

Defined by FCC (Federal Communication commission) United States

## TABLE 5-1 AM BROADCAST STATION TECHNICAL STANDARDS

<table>
<thead>
<tr>
<th>Item</th>
<th>FCC Technical Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assigned frequency, $f_c$</td>
<td>In 10-kHz increments from 540 to 1700 kHz</td>
</tr>
<tr>
<td>Channel bandwidth</td>
<td>10 kHz</td>
</tr>
<tr>
<td>Carrier frequency stability</td>
<td>±20 Hz of the assigned frequency</td>
</tr>
<tr>
<td>% modulation</td>
<td>Maintain 85-95%; max.:</td>
</tr>
<tr>
<td>Noise</td>
<td>At least 45 dB below 100% modulation in the band 30 Hz to 20 kHz</td>
</tr>
<tr>
<td>Maximum power licensed</td>
<td>50 kW</td>
</tr>
</tbody>
</table>
Applications of AM: Superheterodyne Receiver

- Commercial AM radios use the frequency band 535-1605 kHz for transmission.
- The carrier frequency range is from 540-1600 kHz with 10-kHz spacing.
- The message signal is limited to bandwidth of 5 kHz.

**Functions:**

1. **Carrier frequency Tuning:** to select the desired signal *(Antenna + RF + Mixer)*
2. **Filtering:** Separating the desired signal from other modulated signals *(RF coarse tuning + IF fine tuning)*
3. **Demodulation:** Envelope detector.
4. **Amplification:** To compensate for the loss of signal power *(IF section and Loudspeaker)*
Applications of AM: Superheterodyne Receiver

The output of the mixer is the intermediate frequency which is given by: \( f_{IF} = f_{LO} \pm f_c \)

We consider only \( f_{IF} = f_{LO} - f_c \) and the other frequency \( f_{IF} = f_{LO} + f_c \) is considered as image frequency. This image frequency is rejected using band pass filter as shown in the second block diagram.
Applications of AM: Superheterodyne Receiver

- The RF section is tuned to the carrier frequency.
- The combination of the RF and the local oscillator provide a heterodyne function: Converting the signal to the fixed Intermediate Frequency (IF).

**Heterodyning:**
Changing the incoming carrier frequency (which changes with the selected channel) to a fixed smaller frequency $f_{IF}$.

**Result:**
Seems as if the message signal was modulated using $f_{IF}$.

**Reason:**
Adequate selectivity.
- It is difficult to design a filter when $f_c$ is tunable (It takes different values for different broadcasting stations)
## Superheterodyne Receiver: Heterodyne Function

### Table 2.3  Typical frequency parameters of AM and FM radio receivers

<table>
<thead>
<tr>
<th></th>
<th>AM Radio</th>
<th>FM Radio</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF carrier range</td>
<td>0.535–1.695 MHz</td>
<td>88–108 MHz</td>
</tr>
<tr>
<td>Midband frequency of IF section</td>
<td>0.455 MHz</td>
<td>10.7 MHz</td>
</tr>
<tr>
<td>IF bandwidth</td>
<td>10 kHz</td>
<td>200 kHz</td>
</tr>
</tbody>
</table>
Superheterodyne Receiver: IF Section

- One or more stages of tuned amplification.
- Provides most of the amplification and selectivity of the receiver.
- The RF bandwidth is considered relatively large. The small bandwidth of the IF section provides more rejection to interference.
- IF selective-band amplification provides extra discrimination against interfering signals.

Super-heterodyne Receiver: Demodulator

- Provide the main difference between different modulation techniques.
- The output of the demodulator is the message signal that is applied directly to the speakers.
- When coherent detection is required the demodulator must be coherent type.
Advantages of Superheterodyne Receiver

• It reduces the signal from very high frequency sources where ordinary components wouldn't work (processing in low frequency is easier).

• It allows many components to operate at a fixed frequency (IF section) and therefore they can be optimized or made more inexpensively.

• It can be used to improve signal isolation (interference rejection).
AM Properties Summary

- Advantages:

*Easy* to realize, simple circuits for both modulation and de-modulation and thus low cost.

- Disadvantages:

2. *Waste* of bandwidth