Formula Sheet

MVU:

$$\hat{\boldsymbol{\theta}}_{MVU} = \left(\mathbf{H}^T \mathbf{H}\right)^{-1} \mathbf{H}^T \mathbf{x}$$

$$\begin{aligned}\hat{\boldsymbol{\theta}}_{MVU} &= \left(\mathbf{H}^T \mathbf{H}\right)^{-1} \mathbf{H}^T \mathbf{x}' \\ &= \left(\mathbf{H}^T \mathbf{H}\right)^{-1} \mathbf{H}^T (\mathbf{x} - \mathbf{s})\end{aligned}$$

$$\hat{\theta} \sim \mathcal{N}(\boldsymbol{\theta}, \sigma^2(\mathbf{H}^T\mathbf{H})^{-1})$$

BLUE algorithm:

$$\hat{\theta} = (\mathbf{H}^T\mathbf{C}^{-1}\mathbf{H})^{-1}\mathbf{H}^T\mathbf{C}^{-1}\mathbf{x} \quad \text{and} \quad \mathbf{C}_{\hat{\theta}} = (\mathbf{H}^T\mathbf{C}^{-1}\mathbf{H})^{-1}$$

LS:

$$\hat{\theta} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{x}$$

CRLB:

$$CRLB = \frac{1}{-E\left\{\dfrac{\partial^2 p(\mathbf{x};\theta)}{\partial\theta^2}\right\}\Bigg|_{\theta=truevalue}}$$

ML:

$$\hat{\theta} = \arg\max_{\theta} p(\mathbf{x};\theta)$$

MMSE:

$$\text{The estimator is:} \quad \begin{aligned}\hat{\theta} &= E\{\theta \mid \mathbf{x}\} \\ &= \int \theta\, p(\theta \mid \mathbf{x}) d\theta\end{aligned}$$

In Matrix Form:

$$E(\boldsymbol{\theta}|\mathbf{x}) = \boldsymbol{\mu}_\theta + \mathbf{C}_\theta\mathbf{H}^T(\mathbf{H}\mathbf{C}_\theta\mathbf{H}^T + \mathbf{C}_w)^{-1}(\mathbf{x} - \mathbf{H}\boldsymbol{\mu}_\theta)$$

$$\mathbf{C}_{\theta|x} = \mathbf{C}_\theta - \mathbf{C}_\theta\mathbf{H}^T(\mathbf{H}\mathbf{C}_\theta\mathbf{H}^T + \mathbf{C}_w)^{-1}\mathbf{H}\mathbf{C}_\theta.$$

MAP:

$$\hat{\theta} = \arg\max_{\theta} p(\mathbf{x}|\theta) \qquad\qquad p(\theta|\mathbf{x}) = \frac{p(\mathbf{x}|\theta)p(\theta)}{p(\mathbf{x})}$$

Estimate using SVD:

$$\hat{\boldsymbol{\theta}}_{LS} = \mathbf{V}\boldsymbol{\Sigma}^{-1}\mathbf{U}^T\mathbf{x} \quad \text{where} \quad \mathbf{U} = \mathbf{H}\mathbf{V}\boldsymbol{\Sigma}^{-1}$$

RLS:

Estimator Update :

$$\hat{\theta}[n] = \hat{\theta}[n-1] + \mathbf{K}[n]\left(x[n] - \mathbf{h}^T[n]\hat{\theta}[n-1]\right)$$

where

$$\mathbf{K}[n] = \frac{\Sigma[n-1]\mathbf{h}[n]}{\sigma_n^2 + \mathbf{h}^T[n]\Sigma[n-1]\mathbf{h}[n]}$$

Covariance Update :

$$\Sigma[n] = (\mathbf{I} - \mathbf{K}[n]\mathbf{h}^T[n])\Sigma[n-1]$$

To start the algorithm:

$$\hat{\theta} = (\mathbf{H}^T\mathbf{C}^{-1}\mathbf{H})^{-1}\mathbf{H}^T\mathbf{C}^{-1}\mathbf{x} \quad \text{and} \quad \mathbf{C}_{\hat{\theta}} = (\mathbf{H}^T\mathbf{C}^{-1}\mathbf{H})^{-1}$$

$$\hat{\theta}[n-1] = \Sigma[n-1]\mathbf{H}^T[n-1]\mathbf{C}^{-1}[n-1]\mathbf{x}[n-1]$$

$$\text{with} \quad \Sigma[n-1] = (\mathbf{H}^T[n-1]\mathbf{C}^{-1}[n-1]\mathbf{H}[n-1])^{-1}$$

LRT:

$$\Lambda(\mathbf{R}) = \frac{p_{\mathbf{r}|H_1}(\mathbf{R}|H_1)}{p_{\mathbf{r}|H_0}(\mathbf{R}|H_0)}$$

$$P_F = \int_{Z_1} p_{\mathbf{r}|H_0}(\mathbf{R}|H_0)\, d\mathbf{R},$$

$$P_D = \int_{Z_1} p_{\mathbf{r}|H_1}(\mathbf{R}|H_1)\, d\mathbf{R},$$

$$P_M = \int_{Z_0} p_{\mathbf{r}|H_1}(\mathbf{R}|H_1)\, d\mathbf{R}$$

Trigonometric identities

$$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$$
$$\cos\theta = \tfrac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$$
$$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta - \sin^2\theta = \cos(2\theta)$$
$$\cos^2\theta = \tfrac{1}{2}[1 + \cos(2\theta)]$$
$$\sin^2\theta = \tfrac{1}{2}[1 - \cos(2\theta)]$$
$$2\sin\theta\cos\theta = \sin(2\theta)$$
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$
$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$
